314320

25226

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

Marks

1. Attempt any FIVE of the following : **10**

(a) If $f(x, y) = 3x + 4xy$, find $\frac{\partial f}{\partial x}$.

(b) If $f(x, y) = e^{x \cdot y}$, find $\frac{\partial^2 f}{\partial x^2}$.

(c) Find the eigen value of the matrix,

$$A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$$

(d) Find the rank of the matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(e) If the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 2p\hat{k}$ are perpendicular to each other. Find the value of P.

(f) If $\vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, find $|\vec{a}|$.

(g) Construct backward difference table for the following data :

X	0	1	2	3	4	5
Y	1	3	7	13	21	31

2. Attempt any THREE of the following : **12**

(a) If $f(x, y) = 4x^2y^3 + 6x^3y^2$, find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$.

(b) Find the rank of matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$.

(c) Discuss the maxima and minima of the function $f(x, y) = 3x^2 - y^2 + x^3$.

(d) Reduce the matrix to Normal form and hence find its rank of $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

3. Attempt any THREE of the following : **12**

(a) Using the method of Lagrange's multiplier, find the maximum value of the function $f(x, y) = xy$

Subject to constraint $\phi(x, y) = x + y - 500 = 0$.

(b) Solve the following system of equations :

$x - y + 2z = 3$, $x - 2y + 3z = 5$, $3x - 4y - 5z = -13$

(c) Find eigen values and eigen vectors of the matrix A

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

(d) Find the angle between the vectors, $\bar{a} = 2\bar{i} + 2\bar{j} + \bar{k}$ and $\bar{b} = 3\bar{i} + 6\bar{j} + 3\bar{k}$.

4. Attempt any THREE of the following : **12**

(a) Find area of parallelogram $\left| \bar{a} \times \bar{b} \right|$ formed by two vectors $2\bar{i} + \bar{j}$ and $\bar{j} + 3\bar{k}$.

(b) Find the direction cosines of the vector $\bar{a} = 3i + 4j + 12k$.

(c) If $\bar{a} = 5i - 5j - 7k$, $\bar{b} = 6i + 2j - 2k$, $\bar{c} = 10i - j + 11k$, then find $\bar{a} \cdot (\bar{b} \times \bar{c})$.

(d) Evaluate $\int_0^1 x^3 \, dx$ by using trapezoidal rule. Given that

x	0	0.2	0.4	0.6	0.8	1.0
f(x)	0	0.008	0.064	0.216	0.512	1.000

(e) Find the maximum value of

$z = 7x + 11y$

Subject to constraints

$3x + 5y \leq 26$

$5x + 3y \leq 30$

$x \geq 0, y \geq 0$

5. Attempt any TWO of the following : **12**

(a) (i) Find the maximum and minimum values of $f(x, y) = x^3 - y^2 - 3x$.

(ii) Verify Euler theorem for the following function $f(x, y) = x^3 + y^3$.

(b) Using Simpson's $\frac{1}{3}^{rd}$ rule, evaluate $\int_0^{\pi/2} \sqrt{\cos x} \cdot dx$. Divide the interval into eight equal sub-intervals.

P.T.O.

(c) An electric company manufactures two models at two different plants. The daily capacity of the first plant is 60 radios and that of second is 75 radios. Each unit of the first model requires 10 pieces of a certain electric component, whereas each unit of the second model requires 8 pieces of the same component. The maximum daily availability of the special component is 800 pieces. The profit per unit of the first model is ₹ 300 and that of the second model is ₹ 200. Formulate the L.P.P. and solve by graphical method.

6. Attempt any TWO of the following : **12**

(a) (i) Test for consistency and solve $5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$.

(ii) Find the inverse of a matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by elementary transformation.

(b) Given :

X	0	1	2	3	4	5
Y	4	8	15	7	6	2

From the above data, find y'(0) and y''(0).

(c) Using Simplex method, solve the LPP,

Maximize $z = 3x + 2y$

Subject to $x + y \leq 4$

$x - y \leq 2$

$x, y \geq 0$